What are Composites?

- The term “COMPOSITE” is derived from the Latin “COMPOSITUS” which comes from “COMPONERE”
- COMPONERE is made up of “COM” and “PONERE” meaning “together” and “to put” respectively.
- The general definition of an engineering composite is:
  “THE COMBINATION OF TWO OR MORE DISSIMILAR MATERIALS THAT ARE STRONGER THAN THE INDIVIDUAL MATERIALS”
- This includes both NATURAL and MAN-MADE composites.

- A more specific definition was given by B. STRONG in his book “Fundamentals of Composites Manufacturing”:
  “The combination of a reinforcement in a matrix material”
- The term composite also means that the materials (matrix and reinforcement) are identifiable at the macroscopic level.

CHASSIS - carbon FIBER composite materials
- It is super lightweight.
- It is super strong.
- It is super stiff.
- It can be easily molded into all kinds of different shapes.
An engineering composite must meet the following criteria:

1. Must contain two or more constituents
2. Processed in a way that the form, distribution and amount of constituents are controlled in a predetermined way
3. Must have unique, useful and superior performance that can be predicted from the properties, amounts and arrangements of constituents using principles of mechanics

What Benefits Do We Get From A Composite?

- Stiffness
- Strength
- Toughness
- Wear Resistance
- Thermal properties
- Low CTE (Coefficient of Thermal Expansion)

Materials Considerations

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal</td>
<td>Metal Matrix Composites (MMC)</td>
</tr>
<tr>
<td>Ceramic</td>
<td>Ceramic Matrix Composites (CMC)</td>
</tr>
<tr>
<td>Polymer</td>
<td>Polymer Matrix Composites (PMC)</td>
</tr>
</tbody>
</table>

- Continuous (long fibres): e.g; SiC, C,
- Discontinuous
  - Short fibres: e.g; AlO, SiC
  - Particulates: e.g; SiC AlO₂
  - Whiskers: e.g; SiC

Figure 3.1: Relative Strength and Stiffness (Compared to Steel) of Various Composites and Aluminum Alloys
Classifications:

- **Composites:**
  - Multiphase material with significant proportions of each phase.
  - **Matrix:**
    - The continuous phase
    - Purpose is to:
      - Transfer stress to other phases
      - Protect phases from environment
    - Classification: MMC, CMC, PMC
  - **Dispersed phase:**
    - Purpose: enhance matrix properties.
    - MMC: increase $\sigma_y$, TS, creep resist.
    - CMC: increase $K_c$.
    - PMC: increase $E$, $\sigma_y$, TS, creep resist.
  - Classification: Particle, fiber, structural

---

Composite Survey

- Reinforcement
  - Properties Dominate
    - Continuous Fibre
    - Particulate
    - Whiskers

- Matrix
  - Properties Dominate

- Classification of Reinforcement
  - Monofilaments
  - Particulate/Whiskers/Staple Fibres

- Reinforced Composites
  - Large-particle dispersion-strengthened
  - Continuous (aligned)
  - Discontinuous (unplied)

- Structural Composites
  - Laminates
  - Sandwich panels
Examples:
- Spheroidite steel: matrix - ferrite (α) (ductile); particles - cementite (Fe₃C) (brittle)
- WC/Co cemented carbide: matrix - cobalt (ductile); particles - WC (brittle, hard)
- Automobile tires: matrix - rubber (compliant); particles - C (stiffer)

Elastic modulus, $E_c$, of composites:
- upper limit: rule of mixtures
  \[ E_c = V_m E_m + V_p E_p \]
- lower limit: 1

Application to other properties:
- Electrical conductivity, $\sigma_e$: Replace $E$ in equations with $\sigma_e$.
- Thermal conductivity, $k$: Replace $E$ in equations with $k$.

Concrete:
- gravel + sand + cement
  - Why sand and gravel? Sand packs into gravel voids
  - Reinforced concrete: Reinforce with steel rod or remesh
  - Prestressed concrete: remesh under tension during setting of concrete, tension release puts concrete under compressive force
  - Post tensioning: tighten nuts to put under tension

Fibers very strong:
- Provide significant strength improvement to material
- Ex: fiber-glass
  - Continuous glass filaments in a polymer matrix
  - Strength due to fibers
  - Polymer simply holds them in place
Composite Survey: Fiber-II

<table>
<thead>
<tr>
<th>Particle-reinforced</th>
<th>Fiber-reinforced</th>
<th>Structural</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber Materials</td>
<td>Whiskers</td>
<td></td>
</tr>
<tr>
<td>- Thin single crystals - large length to diameter ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- graphite, SiN, SiC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- high crystal perfection – extremely strong, strongest known</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- very expensive</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Fibers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Polycrystalline or amorphous</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Generally polymers or ceramics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Ex: $\text{Al}_2\text{O}_3$, Aramid, E-glass, Boron, UHMWPE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Wires</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Metal – steel, Mo, W</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fiber Alignment

- Longitudinal direction
- Transverse direction
- Aligned continuous
- Aligned discontinuous
- Random discontinuous

Composite Survey: Fiber-III

- Aligned Continuous fibers
- Examples:
  - Metal: $\gamma'(\text{Ni}_3\text{Al})-\alpha$(Mo) (ductile)
    - by eutectic solidification;
      matrix: $\alpha$(Mo) (ductile)
  - Ceramic: Glass w/SiC fibers formed by glass slurry
    $E_{\text{fibers}} = 76$ GPa; $E_{\text{matrix}} = 400$ GPa

Composite Survey: Fiber-IV

- Discontinuous, random 2D fibers
- Example: Carbon-Carbon
  - process: fiber/pitch, then burn out at up to 2500°C.
  - uses: disk brakes, gas turbine exhaust flaps, nose cones.
- Other variations:
  - Discontinuous, random 3D
  - Discontinuous, 1D
**Composite Survey: Fiber-V**

- Critical fiber length for effective stiffening & strengthening:
  - Fiber strength in tension
  - Fiber diameter
  - Shear strength of fiber-matrix interface

- Ex: For fiberglass, fiber length > 15 mm needed
- Why? Longer fibers carry stress more efficiently!

**Composite Strength: Longitudinal Loading**

Continuous fibers - Estimate fiber-reinforced composite strength for long continuous fibers in a matrix

- Longitudinal deformation

\[ \sigma_c = \sigma_m V_m + \sigma_f V_f \]

but \[ E_c = E_m V_m + E_f V_f \] longitu(dinal (extensional) modulus

\[ f = \text{fiber} \quad m = \text{matrix} \]

**Composite Strength**

- Estimate of \( E_c \) and TS for discontinuous fibers

- Elastic modulus in fiber direction:

\[ E_c = E_m V_m + KE_f V_f \]

- Estimate when fiber length > 15 \( \frac{\sigma_d}{\tau} \) 

- TS in fiber direction:

\[ (TS)_c = (TS)_m V_m + (TS)_f V_f \]
Particle-Reinforced Composite

- Large particle
  - Particle-matrix interactions cannot be treated on the atomic or molecular level.
  - Most of the composite, particulate is harder and stiffer than matrix
  - These particles tend to restrain movement of the matrix
  - The matrix transfers some of the applied stress to the particles
  - The degree of reinforcement depends on strong bonding at the matrix-particle interface.

- Dispersion-strengthened composites
  - Particles are normally much smaller (diameter: 10-100 nm)
  - Particle-matrix interactions that lead to strengthening occur on the atomic or molecular level.
  - The mechanism of strengthening is similar to precipitation hardening
  - The small dispersed particles hinder or impede the dislocation motions.

Question: Cite the general difference in strengthening mechanism between large-particle and dispersion-strengthened particle-reinforced composites.

- The major difference in strengthening mechanism between large-particle and dispersion-strengthened particle-reinforced composites is that for large-particle the particle-matrix interactions are not treated on the molecular level, whereas, for dispersion-strengthening these interactions are treated on the molecular level.

Fiber-Reinforced Composites

- The design goals: high strength and/or stiffness over weight basis.

- Fiber length:
  - Longer fibers carry stress more efficiently (above critical length)

- Fiber orientation:
  - The properties of a composite having its fibers aligned are highly anisotropy, means dependent on the direction in which they are measured, max strength and reinforcement along the alignment (longitudinal) direction.
  - In transverse, fiber reinforcement is virtually nonexistent.
  - For applications of involving multidirectional usually use discontinuous fibers which are randomly oriented in the matrix material.

<table>
<thead>
<tr>
<th>Fiber Orientation</th>
<th>Stress Direction</th>
<th>Reinforcement Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>All fibers parallel</td>
<td>Parallel to fibers</td>
<td>1</td>
</tr>
<tr>
<td>Fibers randomly and uniformly distributed within a specific plane</td>
<td>Perpendicular to fibers</td>
<td>0</td>
</tr>
<tr>
<td>Fibers randomly and uniformly distributed within three dimensions in space</td>
<td>Any direction in the plane of the fibers</td>
<td>1</td>
</tr>
<tr>
<td>Fibers randomly and uniformly distributed within three dimensions in space</td>
<td>Any direction</td>
<td>1</td>
</tr>
</tbody>
</table>

Question: Cite one desirable characteristic and one less desirable characteristic for each of (1) discontinuous-oriented (aligned), and (2) discontinuous-random fiber-reinforced composites.

Answer: For discontinuous-oriented fiber-reinforced composites one desirable characteristic is that the composite is relatively strong and stiff in one direction; a less desirable characteristic is that the mechanical properties are anisotropic.

For discontinuous and random fiber-reinforced, one desirable characteristic is that the properties are isotropic; a less desirable characteristic is there is no single high-strength direction.

Example

A metal matrix composite is made from a boron (B) reinforced aluminum alloy (figure). To form the boron fiber a tungsten wire (W) (\(r = 10 \mu m\)) is coated with boron, giving a final radius of 75 \(\mu m\). The aluminum alloy is then bonded around the boron fibers, giving a volume fraction of 0.65 for the aluminum alloy. Assuming that the rule of binary mixtures applies also to ternary mixtures, calculate the effective tensile elastic modulus of the composite material under isostrain conditions.

(Given : \(E_W = 410 \text{ GPa}, E_B = 379 \text{ GPa}, \text{ and } E_{Al} = 68.9 \text{ GPa}\))

Solution:

\[
E_C = f_W E_W + f_B E_B + f_{Al} E_{Al}
\]

\[
f_W = \frac{\pi (10 \mu m)^2}{\pi (75 \mu m)^2} \times (0.35) = 6.22 \times 10^{-3};
\]

\[
f_{Al} = 0.65
\]

\[
f_B = \frac{\text{area}_{\text{B fiber}} - \text{area}_{W wire}}{\text{area}_{\text{B fiber}}} \times f_{W+B}
\]

\[
\frac{\pi (75 \mu m)^2 - \pi (10 \mu m)^2}{\pi (75 \mu m)^2} \times (0.35) = 0.344
\]

\[
E_C = (6.22 \times 10^{-3})(410 \text{ GPa}) + (0.344)(379 \text{ GPa}) + (0.65)(68.9 \text{ GPa}) = 178 \text{ GPa}
\]

Note: the tensile modulus (stiffness) of the composite is about 2.5 times that of the unreinforced aluminum alloy.
Matrix Materials for Engineering Composites

<table>
<thead>
<tr>
<th>Material</th>
<th>Specific Gravity</th>
<th>Tensile Strength (GPa)</th>
<th>Specific Strength (GPa)</th>
<th>Modulus of Elasticity (GPa)</th>
<th>Specific Modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphite</td>
<td>2.2</td>
<td>20</td>
<td>0.1</td>
<td>700</td>
<td>318</td>
</tr>
<tr>
<td>Silicon nitride</td>
<td>3.2</td>
<td>(5–7)</td>
<td>1.56 – 2.2</td>
<td>300 – 380</td>
<td>100 – 110</td>
</tr>
<tr>
<td>Aluminum oxide</td>
<td>4.0</td>
<td>(10–20)</td>
<td>2.4 – 5.0</td>
<td>700 – 130</td>
<td>175 – 375</td>
</tr>
<tr>
<td>Silicon carbide</td>
<td>3.2</td>
<td>20</td>
<td>0.23</td>
<td>480</td>
<td>150</td>
</tr>
</tbody>
</table>

**Metallic Fibers**

- High-strength steel
- Molybdenum
- Tungsten

---

**Table 13.4 Characteristics of Several Fiber-Reinforcement Materials**

<table>
<thead>
<tr>
<th>Material</th>
<th>Fibers</th>
<th>Tensile Strength (GPa)</th>
<th>Modulus of Elasticity (GPa)</th>
<th>Specific Modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum oxide</td>
<td>3.95</td>
<td>1.38</td>
<td>0.35</td>
<td>279</td>
</tr>
<tr>
<td>Aramid (Kevlar 49)</td>
<td>1.44</td>
<td>3.6 – 4.1</td>
<td>2.5 – 2.85</td>
<td>121</td>
</tr>
<tr>
<td>Carbon</td>
<td>1.78 – 2.15</td>
<td>1.5 – 4.6</td>
<td>0.70 – 2.70</td>
<td>229 – 714</td>
</tr>
<tr>
<td>E-Glass</td>
<td>2.58</td>
<td>3.4</td>
<td>1.34</td>
<td>25</td>
</tr>
<tr>
<td>Boron</td>
<td>2.57</td>
<td>3.0</td>
<td>1.40</td>
<td>800</td>
</tr>
<tr>
<td>Silicon carbide</td>
<td>3.0</td>
<td>3.0</td>
<td>1.30</td>
<td>400</td>
</tr>
<tr>
<td>SiC/MW/MWPE (Spectra 900)</td>
<td>0.97</td>
<td>2.0</td>
<td>2.68</td>
<td>17</td>
</tr>
</tbody>
</table>

---

**Matrix Materials for Engineering Composites**

- **Thermoplastic resins**
  - Low cost processing
- **Thermosets**
  - Tough, Formable
  - Low thermal and solvent resistant
  - High cost to process
- **Carbon**
  - Very high temperature applications
  - Very high cost to process
- **Laminates**
  - Thermal resistant
  - Electrical and thermal
  - Ready with most thermosets
- **Sheetmetal**
  - Oxidation resistant
  - Heat
- **Billets**
  - High temperature strength
  - Corrosion resistant
- **Cables**
  - Continuous and constant thermal expansion
- **Ceramics**
  - Continuous and temperature resistant
  - Thermal and expansion resist
- **Ceramic**
  - Very high temperature
  - Oxidation

---

**Metal Matrix Composites**

- MMC’s consist of a low density metal reinforced with a ceramic material
  - **Aluminium (Al), Magnesium (Mg), and Titanium (Ti)**
- The most common metal alloys used as matrix are:
  - Continuous: long fibres
  - Discontinuous: particulates or whiskers
  - E.g. Continuous B fiber – aluminum alloy matrix composite
Metal Matrix Composites

**Mechanical Properties of Metal-Matrix Composite Materials**

<table>
<thead>
<tr>
<th></th>
<th>Tensile Strength</th>
<th>Elastic Modulus</th>
<th>Strain to failure, %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Continuous-fiber MMCs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al 2024-T6 [60% SiC]</td>
<td>1458</td>
<td>211</td>
<td>220</td>
</tr>
<tr>
<td>Al 6061-T6 (50% SiC)</td>
<td>1417</td>
<td>235</td>
<td>231</td>
</tr>
<tr>
<td>Al 6061-T6 (20% SiC)</td>
<td>1402</td>
<td>212</td>
<td>204</td>
</tr>
<tr>
<td><strong>Discontinuous-fiber MMCs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al 2024-T6 (20% SC)</td>
<td>650</td>
<td>94</td>
<td>127</td>
</tr>
<tr>
<td>Al 6061-T6 (20% SC)</td>
<td>480</td>
<td>70</td>
<td>115</td>
</tr>
<tr>
<td><strong>Particulate MMCs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al 2124 (20% SC)</td>
<td>552</td>
<td>80</td>
<td>103</td>
</tr>
<tr>
<td>Al 6061 (20% SC)</td>
<td>496</td>
<td>72</td>
<td>103</td>
</tr>
<tr>
<td>No reinforcement</td>
<td>465</td>
<td>66</td>
<td>71</td>
</tr>
</tbody>
</table>

** MMC offer:**
- Higher specific stiffness
- Higher operating temperatures
- Greater wear resistance
- Possibility to tailor the properties for a specific application.

** MMC disadvantages include:**
- Higher cost of materials and processing
- Lower ductility and toughness

- The reinforcement type determines the mechanical properties, cost and performance of the composite material produced

1. Continuous Fibres:
   - Good properties
   - Expensive
   - Difficult to process
   - No secondary processing
   - Have larger defense demand, but little commercial demand

2. Discontinuous Fibres:
   - Poor properties
   - Lower cost
   - Easier to process
   - Secondary processing possible
   - Have considerable defense and commercial demand

- Currently the focus is on two main types of MMC’s:

1. High performance composites reinforced with expensive fibres which require the use of expensive processing techniques (military and space applications)
2. Low cost and low performance composites reinforced with relatively cheaper particulates (commercial applications)

**Applications of MMC’s**

1. **Transport:** Automotive, Aerospace (Al matrix-B fiber, TiAl matrix-SiC fiber), and Marine
2. **Sport**
• MMC’s can be manufactured by several production processes which can be divided into:

1. Primary processing methods
   • Liquid State Processes
     - Squeeze casting: Molten metal is injected into a form with fibers preplaced inside it.
     - Stir Casting: Discontinuous reinforcement is stirred into molten metal, which is allowed to solidify.
   • Solid State Processes
     - Powder Metallurgy, Diffusion Bonding

2. Secondary processing methods
   • Extrusion

### Table 4.5—MMC Manufacturing Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
<th>Fiber Type</th>
<th>Process Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary methods</td>
<td>Casting, including stir casting, squeeze casting</td>
<td>Continuous fiber, discontinuous fiber, whiskers, particulates</td>
<td>Liquid state, solid state</td>
</tr>
<tr>
<td>Secondary methods</td>
<td>Extrusion, sintering, pressing, diffusion bonding</td>
<td>Continuous fiber, discontinuous fiber, whiskers, particulates</td>
<td>Liquid state, solid state</td>
</tr>
<tr>
<td>Powder Metallurgy</td>
<td>Powder consolidation, sintering, bonding</td>
<td>Continuous fiber, discontinuous fiber, whiskers, particulates</td>
<td>Solid state</td>
</tr>
<tr>
<td>Diffusion Bonding</td>
<td>Sintering, bonding</td>
<td>Continuous fiber, discontinuous fiber, whiskers, particulates</td>
<td>Solid state</td>
</tr>
<tr>
<td>Spray Deposition</td>
<td>Spraying, deposition</td>
<td>Continuous fiber, discontinuous fiber, whiskers, particulates</td>
<td>Liquid state</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>Fiber Type</th>
<th>Process Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monofilaments</td>
<td>High</td>
</tr>
<tr>
<td>Continuous Fiber</td>
<td>Diffusion Bonding</td>
</tr>
<tr>
<td>Whiskers</td>
<td>Powder Metallurgy</td>
</tr>
<tr>
<td>Particulates</td>
<td>Spray Deposition</td>
</tr>
<tr>
<td>Liquid Metal</td>
<td>Low</td>
</tr>
</tbody>
</table>
Stir Casting of Particulate Reinforced MMC's

- This method is currently the most common commercial technique of producing Al-based MMC reinforced with ceramic particulates.
- It involves mixing of particulates into a liquid or semi-solid metal matrix.
- When the matrix is in the semi-solid condition the method is generally known as "compocasting".
- Issues in Liquid State Processing of MMC's:
  - Wetting between liquid matrix (liquid) and reinforcement (solid)
  - Interfacial reaction between matrix and reinforcement.

Cast MMC's have several characteristics:

- Stir casting is simple and cheap.
- Wetting between matrix and reinforcement is ensured by pretreatment of the particulates (heat treatment or coating).
- High volume production possible.
- Chemical reaction at the matrix / reinforcement interface is a problem.
Uniform distribution of SiC particles

**Cast Particulate Al-SiC MMC**

**Squeeze Casting of MMC’s**

- Liquid Metal
- Preform
- MMC

**Powder Metallurgy of MMC’s**

- Mix Raw Materials
  - SiC
  - Al-Powder
- Cold Compaction + Sintering
- Secondary Process (Extrusion)
- MMC Billet

**MMC produced by PM is characterised by:**

1. Unique MMC material chemistry, with very fine microstructure and uniform distribution of reinforcement
2. 10 – 40 % of reinforcement is possible
3. High properties: Strength, Ductility, Toughness, Fatigue resistance
4. Little chemical reaction at the matrix / reinforcement interface.
5. High quality products (aerospace applications)
6. Higher cost
**Diffusion Bonding of MMC’s**

MMC’s produced by Diffusion Bonding

**Vapour Deposition of MMC’s**

Ti-5Al-5V alloy/80vol% SiC fibres consolidated by hot pressing or HIP

**Co-Spray of MMC’s**

**Applications of MMC’s**

<table>
<thead>
<tr>
<th>Reinforcement</th>
<th>Aerospace</th>
<th>Automotive</th>
<th>Advantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous Fibres</td>
<td>Fibre, compressor blades, aircraft structure, engine components</td>
<td></td>
<td>High thrust to weight ratio, high stiffness, low density, controlled CTE</td>
</tr>
<tr>
<td>Discontinuous Reinforcement</td>
<td>Wing panels, precision components, engine components</td>
<td>Piston, connecting rods, bearings, cylinder liners, brake parts, drive shafts,</td>
<td>Wear resistance, low cost, elevated temperature properties, fatigue strength</td>
</tr>
<tr>
<td>Particulate and whiskers</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Applications: Reinforcement Aerospace Automotive Advantage

- Continuous Fibres: Fin, compressor blades, aircraft structure, engine components
- Discontinuous Reinforcement: Wing panels, precision components, engine components
- Particulate and whiskers: Piston, connecting rods, bearings, cylinder liners, brake parts, drive shafts

Advantages:
- High thrust to weight ratio
- High stiffness
- Low density
- Controlled CTE
### Mechanical Behaviour of Composites

- Mechanical behaviour of composites depend on several factors:
  - Stress – Strain behaviors of matrix and reinforcement
  - Phase volume fractions
  - Direction in which the stress is applied

1. **Stress strain behaviour of composite**

- Properties of a composite represent an average of the properties of the matrix and reinforcement
- However, this also depends on the geometry

#### Properties of a composite

- If the composite, matrix and reinforcement are elastic $E = \frac{F}{A}$
- Strain behaviors of matrix and reinforcement
- Dividing by $A$

#### Isostrain

- The ratio of the load carried by the fibers to that carried by the matrix is:

$$ P_c \frac{A_c}{A_m} = \frac{E_f \varepsilon_c}{E_m \varepsilon_m} $$

#### Direction of Applied Stress (Continuous Fibres):

**i) Loading Parallel to Reinforcing Fibres - Isostrain**

- If the matrix is intimately bonded to the fibres, the strain of both matrix and fibres is the same, but the load carried by the composite is equal to the loads carried by the matrix and fibers (reinforcement)

$$ F = F_m + F_r $$

- Since $\sigma = \frac{F}{A}$

$$ \sigma_c A_c = \sigma_m A_m + \sigma_r A_r $$

### Under Isostrain conditions

- If the composite, matrix and reinforcement are elastic $E = \frac{F}{A}$

$$ E = E_m V_m + E_f V_f $$

or

$$ \varepsilon = \varepsilon_m (1 - V_f) + \varepsilon_f V_f $$

- Another important parameter which is significant to the contribution of the reinforcement to the composite modulus is the fraction of the total composite load, $P_c$

$$ P_r \frac{A_c}{A_m} = \frac{E_f \varepsilon_c}{E_m \varepsilon_m} $$

- The ratio of the load carried by the fibers to that carried by the matrix is:

$$ \frac{P_r}{P_m} = \frac{E_f V_f}{E_m V_m} $$
2. Direction of Applied Stress (Continuous Fibres):

i) Loading Normal to Reinforcing Fibres - Isostress

- Under these conditions:
  \[ \sigma_c = \sigma_m = \sigma_r \]
- The strain of the composite is:
  \[ \varepsilon_c = \varepsilon_m V_m + \varepsilon_r V_r \]
- But since \( \varepsilon = \varepsilon_f \),

\[
\frac{\sigma}{E_c} = \frac{\sigma}{E_m} V_m + \frac{\sigma}{E_r} V_r
\]

- Dividing by \( \sigma \),

\[
\frac{1}{E_c} = \frac{V_m}{E_m} \frac{V_r}{E_r}
\]

- This also gives:

\[
E_c = \frac{E_m V_r + V_m E_r}{(1 - V_f) E_r + V_r E_m}
\]