The mechanical behaviour of a material can be described largely in terms of the materials properties that govern plastic deformation and fracture.

Knowledge and understanding of the relevant properties is the first step toward improving these properties and/or developing new materials with superior properties.

Plastic deformation occurs by shear, and at much lower shear stresses (or tensile yield stresses) than the theoretical shear stress as a result of dislocation slip.

Fortunately, a number of strengthening mechanisms exist, whereby the yield strength of ductile materials can be enhanced considerably.

**WHY MATERIALS FRACTURE?**

- Fracture is the catastrophic break-up of the structure into two or more pieces, usually caused by a structural defect or a crack.

- Due to service loading the crack may develop and grow slowly in size, reducing the strength of the material.

- As the crack grows in length, the strength decreases until it becomes so low that the service loads cannot be carried any more, and fracture occurs.

- We need to understand how materials fail.

- Examples:
  1. Liberty ships produced during WW2: steel became brittle in north Atlantic sea
  2. Titanic: ductile – brittle transition in cold water
THE PROCESS OF FRACTURE

- Three stages of fracture:
  1. Crack initiation. Crack will initiate at the point of stress concentration (scratches, fillets, threads, dents) when the internal stress cannot cope with the applied stress.
  2. Crack propagation. The applied force will propagate the crack.
     - Stage I: propagates very slowly along crystallographic planes of high shear stress and may constitute either a large or small fraction of the fatigue life of a specimen.
     - Stage II: the crack growth rate increases and changes direction, moving perpendicular to the applied stress.
  3. Fracture. Crack that exceeds a critical size will cause fracture to occur.

- Common causes of fracture are:
  - Incorrect material selection
  - Poor design
    - Holes. Either incorporated in the design for aesthetic purposes or otherwise. Could also be due to macroscopic defect within the materials.
  - Use of new design or material, which produces unexpected results
  - Surface damage. Due to mechanical load such as scratching or surface roughness.
  - Environment. High temperature and pressure, as well as corrosive environment increase the cause of fracture.

There are two fracture modes:

- Ductile failure (high energy): occurs with plastic deformation. Often the crack will only propagate with additional applied stress (stable crack).
- Brittle failure (low energy): no plastic deformation. (crack is unstable)
  - Catastrophic

Ductile fracture

Brittle fracture
FRACTURE

- Classification:
  Fracture behavior: Very Ductile → Moderately Ductile → Brittle
  %AR or %EL: Large → Moderate → Small

- Ductile fracture is desirable!

- Evolution to failure:
  necking → void nucleation → void growth and linkage → shearing at surface → fracture

- Resulting fracture surfaces (steel):
  particles serve as void nucleation sites.

- Ductile failure:
  --one piece
  --large deformation

- Brittle failure:
  --many pieces
  --small deformation

- Ext: Failure of a pipe
  Evolution to failure:
  necking → void nucleation → void growth and linkage → shearing at surface → fracture

- Resulting fracture surfaces (steel):
  particles serve as void nucleation sites.

- Figures from V.J. Colangelo and F.A. Heiser, Analysis of Metallurgical Failures (2nd ed.), Fig. 4.1(a) and (b), p. 66. John Wiley and Sons, Inc., 1987. Used with permission.

- Fracture surface of tire cord wire loaded in tension. Courtesy of F. Roehrig, CC Technologies, Dublin, OH. Used with permission.

- Cup-and-cone fracture

- Brittle fracture

Adapted from Fig. 8.1, Callister 6e.

Adapted from Fig. 8.3, Callister 7e.
In order to prevent fracture, we must know how and under what conditions materials fail.

Designing a fracture control plan requires knowledge of the structural strength as it is affected by cracks, and the time involved for cracks to grow to a dangerous size.

Meaning that cracks must be prevented from growing to a size at which the strength would drop below the acceptable limit. In order to determine which size of cracks is admissible one must be able to:

1. Calculate the structural strength affected by cracks
2. Calculate the time in which a crack grows to a permissible size.

Fracture mechanics should be able to provide answers to such questions.
1. Effect of Cracks and Notches (stress concentration)

- Flaws or defects such as: notches and cracks give rise to a stress concentration (local region where the stresses are higher than the nominal or average stress).

- Consider Figure 1a, there is no crack (or notch), the flow lines are straight and the load is uniform (load is evenly distributed).

- If the load path is interrupted by a cut (notch or crack), the flow lines must go around this cut within a short distance as shown in Figure 3b.

- At the tip of the cut the flow lines are closely spaced: more load is flowing through a smaller area which means higher stress: there is **STRESS CONCENTRATION AT THE CRACK TIP**

2. Stress Concentration factor

- At the region near the notch or crack tip, the stress is higher than the average value and is called stress raiser or stress concentrator

- The stress concentration is expressed by a theoretical stress concentration factor, $K_c$, described as the ratio of the maximum stress to the nominal or applied stress:

$$K_c = \frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}}$$
Cracks propagate due to sharpness of crack tip
- A plastic material deforms at the tip, “blunting” the crack.

Energy balance on the crack
- Elastic strain energy-
  - energy stored in material as it is elastically deformed
  - this energy is released when the crack propagates
  - creation of new surfaces requires energy

As a general rule, blunt notches, e.g. a round hole, produce lower stress, sharp notches cause higher stresses. By analysing a plate containing an elliptical hole, Inglis was able to show that the stress concentration factor is:

$$K_t = \frac{\sigma_{\text{max}}}{\sigma_{\text{mat}}} = 1 + 2 \frac{a}{b}$$  \hspace{1cm} 1.1

Where a and b are as defined in Figure 2. The radius of curvature, $\rho$, of the ellipse is: $\rho = \frac{b}{a}$, so that equation (1.1) can also be written as:

$$K_t = \frac{\sigma_{\text{max}}}{\sigma_{\text{mat}}} = 1 + 2 \frac{a}{b} \sqrt{\frac{b}{a}}$$  \hspace{1cm} 1.2

or

$$\sigma_{\text{max}} = \sigma_{\text{mat}} \left(1 + 2 \frac{a}{b} \sqrt{\frac{b}{a}}\right)$$  \hspace{1cm} 1.3

For ellipse, for example, with $a/b = 3$ ($\rho = a/9$), $K = 1 + 2(3/1) = 7$

When a >> b, equation (1.3) becomes:

$$\sigma_{\text{max}} = 2\sigma \sqrt{\frac{a}{b}}$$  \hspace{1cm} 1.4

Figure 2: high stress concentration in elliptical notch
Theoretical Cohesive Strength

- Strength is due to cohesive forces between atoms. High cohesive forces are related to large elastic modulus and high melting points.

- In Figure 3, the is the resultant of the attractive and repulsive forces between the atoms. ($a_0$ is the equilibrium separation)

- If a tensile load is applied, "r" increases. The maximum in the curve is equal to the cohesive strength of the material where both the repulsive and attractive forces are decreased.

It can be estimated as: (theoretical cohesive strength can be obtained if assumed that the cohesive force curve can be represented by a sine curve

\[
\sigma = \sigma_c \sin \frac{2\pi}{\lambda}
\]

$\sigma_c$ (or $\sigma_{max}$) is the theoretical cohesive strength, $x = a - a_0$ is the atomic displacement with a wave length $\lambda$.

- For small displacements, $\sin x \approx x$, then:

\[
\sigma = \sigma_c \frac{2\pi}{\lambda}
\]

- For brittle elastic solid, Hooke's law is

\[
\sigma = E\varepsilon = \frac{\Delta}{a_0}
\]

- Substituting for $\sigma$ we have (eliminate $x$):

\[
\sigma_c = \frac{a_0 E}{2\pi}\]

- If we assume that $a_0 = \lambda/2$, then

\[
\sigma_c = \frac{E}{\pi}
\]

- When fracture occurs, all the work goes into the creation of two new surfaces; each has a surface energy $\gamma_s$. This work done is equal to the energy required to create the two new fracture surfaces, that is:

\[
U_o = 2\gamma_s
\]
The work done per unit area of surface in creating the fracture is the area under the stress-displacement curve.

\[ U_s = \int \sigma \sin \frac{\lambda \sigma}{\pi} dx = \frac{\lambda \sigma}{\pi} \]  

1.11

But this energy is equal to the energy required to create the two new fracture surfaces,

\[ U_s = 2\gamma_c \]  

So,

\[ \gamma_c = \frac{\lambda \sigma}{2\pi} \]  

1.12

or

\[ \lambda = \frac{2\gamma_c}{\sigma} \]  

Substituting 1.12 into eq. 1.8 gives

\[ \sigma_t = \sqrt{\frac{2\gamma_c}{\rho}} \]  

1.13

Metals deform plastically which causes an initially sharp crack to blunt.

In the absence of plastic deformation (brittle fracture), the minimum radius a crack tip can have is on the order of the atomic radius

The sharpest possible crack should be when \( \rho = a_0 \) (atomic displacement)

Hence,

\[ \sigma_{\text{max}} = 2\sigma_{\text{nom}} \sqrt{\frac{a}{a_0}} = 2\sigma_{\text{nom}} \sqrt{\frac{a}{\rho}} \]  

1.14

Assuming \( \sigma_{\text{nom}} = \sigma_e \)

\[ 2\sigma_{\text{nom}} \sqrt{\frac{a}{\rho}} = \frac{2\sigma_e}{\rho} \]  

1.15

Solving for \( \sigma_{\text{fracture}} \)

as half crack length

\[ \sigma_f = \sqrt{\frac{2\gamma_c}{a}} \]  

1.16

The presence of flaws or cracks is responsible for the lower than ideal fracture strength of engineering materials.

In Figure 2, the maximum stress at the crack tip \( \sigma_{\text{max}} \) is given by equation (1.4) as (for \( a >> b \)):

\[ \sigma_{\text{max}} = 2\sigma_{\text{nom}} \sqrt{\frac{a}{\rho}} \]  

The equation predicts an infinite stress at the tip of a sharp crack where \( \rho = 0 \).

However, no material can withstand an infinite stress. This motivated Griffith to develop a fracture theory based on energy rather than local stress.

Griffith’s criterion/theory

- Fracture mechanics was invented during World War I by English aeronautical engineer, A. A. Griffith, to explain the failure of brittle materials.

- Griffith’s work was motivated by two contradictory facts:

  - The stress needed to fracture bulk glass is around 100 MPa (15,000 psi).
  - The theoretical stress needed for breaking atomic bonds is approximately 10,000 MPa (1,500,000 psi).

- A theory was needed to reconcile these conflicting observations. Also, experiments on glass fibers that Griffith himself conducted suggested that the fracture stress increases as the fiber diameter decreases. Hence the uniaxial tensile strength, which had been used extensively to predict material failure before Griffith, could not be a specimen-independent material property. Griffith suggested that the low fracture strength observed in experiments, as well as the size-dependence of strength, was due to the presence of microscopic flaws (cracks) in the bulk material.
The first explanation of the discrepancy between the observed fracture strength of crystals and the theoretical cohesive strength was proposed by Griffith.

Griffith's theory in its original form is applicable only to a perfectly brittle material such as glass. However, the Griffith's ideas have had great influence on the thinking about the fracture of metals. Griffith proposed that a brittle material contains a population of fine cracks which produce a stress concentration of sufficient magnitude so that the theoretical cohesive strength is reached in localized regions at a nominal stress which is well below the theoretical value. When one of the cracks spreads into a brittle fracture, it produces an increase in the surface area of the sides of the crack. This requires energy to overcome the cohesive force of the atoms (increase in surface energy).

Griffith established the following criterion for the propagation of a crack:
- A crack will propagate when the decrease in elastic strain energy is at least equal to the energy required to create the new crack surface.
- The crack must be energetically favourable
- A mechanism for crack propagation must be available
- Energy is required to create fracture surfaces, which is provided by the release of elastic strain energy due to crack growth.

Consider the crack model as shown in the figure. The stress distribution for an elliptical crack was determined by Inglis.
- A decrease in strain energy results from the formation of crack.
- Energy released due to crack growth or the elastic strain energy per unit of plate thickness is equal to \( U_e \) (also can be written as \( U_f \)):

\[
U_e = -\frac{\sigma^2}{2} \pi a^2
\]

Substituting \( \gamma \), in 1.13 into 1.10, gives
- Negative sign is used because growth of the crack releases elastic strain energy.

Energy is required to create fracture surfaces, \( U_f \), or can be written as \( U_f \) (by considering 2a crack length)
- According to Griffith, the crack will propagate under a constant stress if a small increase in crack length produces no change in the total energy of the system; the increase surface energy is compensated by a decrease in elastic strain energy.
- The total energy is:

\[
U = U_f - U_e
\]
The total energy is a function of crack length

\[
\frac{dU}{da} = \frac{d}{da} \left( 4a \gamma_s - \frac{ma^2}{E} \right)
\]

\[
\frac{dU}{da} = 0 \quad \text{therefore} \quad \frac{dU}{da} = \frac{dU}{d\beta}
\]

\[
2\gamma_s = \frac{ma^2}{E}
\]

\[
\sigma_f = \left( \frac{2 \gamma_s}{\pi a} \right)^{1/2}
\]

Example 1
Fracture stress for a brittle material with the following properties:
\( E = 100 \ \text{GPa}, \ \gamma_s = 1 \ \text{J/m}^2, a_o = 0.25 \ \text{nm}, \)
Given, half crack length, \( a = 10^4 a_o \)

Solution:
\[
\sigma_f = \left( \frac{E \gamma_s}{4a} \right)^{1/2} = \left( \frac{100 \times 10^9 \times 1}{4 \times 2.5 \times 10^{-10}} \right) = 10^5 \ \text{Pa} = 100 \ \text{MPa}
\]
Note that the fracture stress is \( E/1000 \) while the theoretical cohesive strength is \( E/5 \). Thus, we see that a small crack produces a very great decrease in the stress of fracture.

Example 2
Determine the cohesive strength of a silica fiber, if \( E = 95 \ \text{GPa}, \ \gamma_s = 1 \ \text{J/m}^2 \) and \( a_o = 1.6 \ \text{Å} \).

Solution:
\[
\sigma_{\text{max}} = \left( \frac{E \gamma_s}{a_o} \right)^{1/2} = \left( \frac{95 \times 10^9 \times 1}{1.6 \times 10^{-10}} \right)^{1/2} = 24.4 \ \text{GPa}
\]

Example 3
What is the magnitude of the maximum stress that exists at the tip of an internal crack having a radius of curvature of \( 1.9 \times 10^{-4} \ \text{mm} \) and a crack length of \( 3.8 \times 10^{-2} \ \text{mm} \). When a tensile stress of 140 MPa is applied?

Solution:
This problem asks that we compute the magnitude of the maximum stress that exists at the tip of an internal crack.

\[
\sigma_{\text{max}} = 2 \sigma_0 \left( \frac{a}{\rho} \right)^{1/2} = \left( \frac{2}{1.40 \text{MPa}} \right) \left( \frac{3.8 \times 10^{-2} \ \text{mm}}{1.9 \times 10^{-4} \ \text{mm}} \right)
\]

\[
= 2800 \ \text{MPa}
\]
Example 4
If the specific surface energy for Al-oxide is 0.9 J/m², compute the critical stress required for the propagation of an internal crack of length 0.4 mm. (Given, modulus of elasticity for Al-oxide = 393 GPa)

Solution:
We may determine the critical stress required for the propagation of an internal crack in aluminum oxide by taking the value of 393 GPa as the modulus of elasticity, we get

$$\sigma_c = \left( \frac{2E\gamma_s}{\pi a} \right)^{1/2} = 33.6 \text{ MPa}$$